# 10.1 First Order Ordinary Differential Equations

1164. Linear Equations

$$\frac{dy}{dx} + p(x)y = q(x).$$

The general solution is

$$y = \frac{\int u(x)q(x)dx + C}{u(x)},$$

where

$$u(x) = \exp(\int p(x)dx).$$

1165. Separable Equations

$$\frac{dy}{dx} = f(x,y) = g(x)h(y)$$

The general solution is given by

$$\int \frac{dy}{h(y)} = \int g(x)dx + C,$$

or

$$H(y) = G(x) + C$$
.

**CLICK HERE** 

## 1166. Homogeneous Equations

The differential equation  $\frac{dy}{dx} = f(x,y)$  is homogeneous, if the function f(x,y) is homogeneous, that is f(tx,ty) = f(x,y).

The substitution  $z = \frac{y}{x}$  (then y = zx) leads to the separable equation dz

$$x \frac{dz}{dx} + z = f(1,z).$$

### **1167.** Bernoulli Equation

$$\frac{dy}{dx} + p(x)y = q(x)y^{n}.$$

The substitution  $z = y^{1-n}$  leads to the linear equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} + (1-n)p(x)z = (1-n)q(x).$$

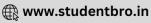
#### 1168. Riccati Equation

$$\frac{dy}{dx} = p(x) + q(x)y + r(x)y^2$$

If a particular solution  $y_1$  is known, then the general solution can be obtained with the help of substitution

$$z = \frac{1}{y - y_1}$$
, which leads to the first order linear equation

$$\frac{dz}{dx} = -[q(x) + 2y_1r(x)]z - r(x).$$



#### **1169.** Exact and Nonexact Equations

The equation

$$M(x,y)dx + N(x,y)dy = 0$$

is called exact if

$$\frac{\partial \mathbf{M}}{\partial \mathbf{y}} = \frac{\partial \mathbf{N}}{\partial \mathbf{x}},$$

and nonexact otherwise.

The general solution is  $\int M(x,y)dx + \int N(x,y)dy = C.$ 

#### **1170.** Radioactive Decay

$$\frac{\mathrm{dy}}{\mathrm{dt}} = -\mathrm{ky}$$
,

where y(t) is the amount of radioactive element at time t, k is the rate of decay.

The solution is

$$y(t) = y_0 e^{-kt}$$
, where  $y_0 = y(0)$  is the initial amount.

## 1171. Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T-S),$$

where T(t) is the temperature of an object at time t, S is the temperature of the surrounding environment, k is a positive constant.

The solution is

$$T(t)=S+(T_0-S)e^{-kt},$$

where  $T_0 = T(0)$  is the initial temperature of the object at time t = 0.

# **1172.** Population Dynamics (Logistic Model)

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{M}\right),$$

Get More Learning Materials Here:

where P(t) is population at time t, k is a positive constant, M is a limiting size for the population.

The solution of the differential equation is

$$P(t)\!=\!\frac{MP_0}{P_0+\!\left(M\!-\!P_0\right)\!e^{-kt}}$$
 , where  $P_0=\!P(0)$  is the initial population at time  $t=0$  .